SOLUTIONS.

# Perth College

**Semester Two Examination, 2016**

**Question/Answer Booklet**

# MATHEMATICS METHODS UNITS 1 AND 2

## Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

In words ______________________

Your name ______________________

## Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| | | | **Total** | 150 | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

**Section One: Calculator-free** **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

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**Question 1** **(4 marks)**

A box contains a total of 500 marker and highlighter pens of various colours, as shown in the table. Some of the marker pens are permanent and the rest are non-permanent.

| | Colour | | | |
|---|---|---|---|---|
| Type of pen | Black | Yellow | Pink | Green |
| Permanent marker | 55 | 83 | 40 | 24 |
| Non-permanent marker | 45 | 67 | 24 | 12 |
| Highlighter | 0 | 50 | 46 | 54 |

A pen is selected at random from the box. Determine the probability that it is

(a) a yellow pen. (1 mark)

$\frac{200}{500}$ ✓

(b) a marker pen. (1 mark)

$\frac{350}{500}$ ✓

(c) a yellow pen or a marker pen. (1 mark)

$\frac{400}{500}$ ✓

(d) a green pen, given that it is a highlighter. (1 mark)

$\frac{54}{150}$ ✓

**Question 2** **(8 marks)**

(a) Determine $f'(x)$ when

(i) $f(x) = 3$. (1 mark)

$f'(x) = 0$ ✓

~~(i)~~ ii $f(x) = 5x^2 - 4x$. (1 mark)

$f'(x) = 10x - 4$ ✓

~~(ii)~~ iii $f(x) = \dfrac{x^3 - 5x}{x}$. (2 marks)

$f(x) = x^2 - 5$ ✓

$f'(x) = 2x$ ✓

(b) Simplify $\lim_{h \to 0} \dfrac{(x+h)^4 - x^4}{h}$ (1 mark)

$= 4x^3$ ✓

(c) Calculate the gradient of the curve $y = 2x^5 - 3x^4$ where $x = -1$. (3 marks)

$\dfrac{dy}{dx} = 10x^4 - 12x^3$ ✓

@ $x = -1$ $\dfrac{dy}{dx} = 10 + 12$ ✓

$= 22$ ✓

**Question 3** **(4 marks)**

$A$ and $B$ are independent events such that $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{4}$. Determine

(a) $P(A \cap B)$. (1 mark)

$= P(A) \times P(B)$

$= \frac{2}{3} \times \frac{1}{4}$

$= \frac{1}{6}$ ✓

(b) $P(B|A)$. (1 mark)

$\frac{1}{4}$ ✓

(c) $P(A \cup B)$. (2 marks)

$= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{3} + \frac{1}{4} - \frac{2}{12}$

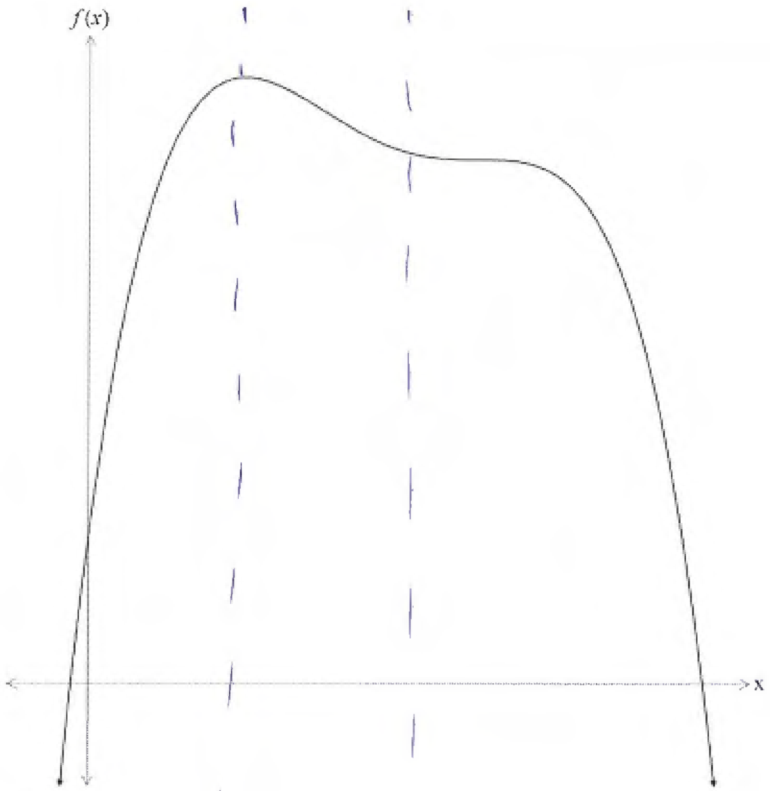
$= \frac{8 + 3 - 2}{12}$

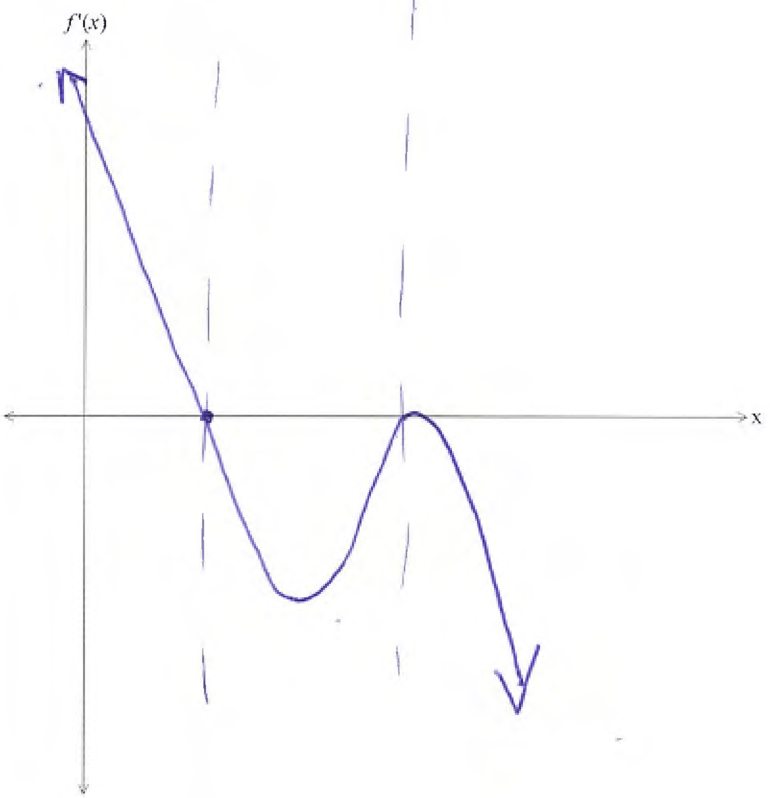
$= \frac{9}{12}$

$= \frac{3}{4}$

**Question 4** **(4 marks)**

The graph of $y = f(x)$ is drawn below. Use this to draw a possible graph of $y = f'(x)$ on the axes provided.



**Question 5** **(13 marks)**

(a) Solve the following equations for $x$:

(i) $3^{x+1} = 9^{1-x}$. (3 marks)

$3^{x+1} = (3^2)^{1-x}$

$x+1 = 2-2x$

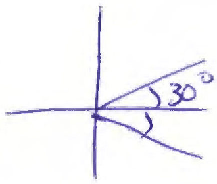
$3x = 1$

$x = 1/3$

(ii) $2\cos x = \sqrt{3}, \quad 0 \le x \le 720°$. (3 marks)

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30°, 330°, 390°, 690°$




-1 per error or missing

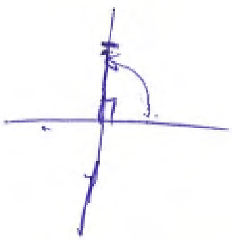
(iii) $\sin 2x \cos x + \cos 2x \sin x = 1, \quad 0 \le x \le \pi$. (3 marks)

$\sin(2x+x) = 1$

$\sin 3x = 1 \quad 0 \le 3x \le 3\pi$

$3x = \pi/2, 5\pi/2,$

$x = \pi/6, 5\pi/6$



(b) The equation $x^3 - x^2 - 14x + 24 = 0$ has $x = 2$ as a solution. Determine all other solutions to the equation. (4 marks)

$$\begin{array}{r} x^2 + x - 12 \\ x-2 \overline{) x^3 - x^2 - 14x + 24} \\ \underline{x^3 - 2x^2} \\ x^2 - 14x \\ \underline{x^2 - 2x} \\ -12x + 24 \\ \underline{-12x + 24} \\ 0 \end{array}$$

$(x-2)(x+4)(x-3) = 0 \therefore x = -4, x = 3$

**Question 6** **(5 marks)**

(a) The expression $(2x-1)^3$ can be expanded to give $8x^3 + ax^2 + 6x - 1$. Show that the value of $a$ is $-12$. (2 marks)

$$(2x-1)^3 = (2x)^3 + 3(2x)^2(-1) + 3(2x)(-1)^2 + (-1)^3$$
$$= 8x^3 - 12x^2 + 6x - 1$$

Term: $3(2x)^2(-1)$ gives $ax^2$
$3 \times 4x^2(-1)$
$= -12x^2$ i.e. $a = -12$

(b) Using the result from (a), or otherwise, determine $f(x)$ if $f'(x) = (2x-1)^3$ and $f(1) = 5$. (3 marks)

$$f'(x) = 8x^3 - 12x^2 + 6x - 1$$
$$f(x) = 2x^4 - 4x^3 + 3x^2 - x + c$$
$$f(1) = 5 \therefore 2 - 4 + 3 - 1 + c = 5$$
$$\therefore c = 5$$
$$f(x) = 2x^4 - 4x^3 + 3x^2 - x + 5$$

**Question 7** **(7 marks)**

The first three terms, in order, of a geometric sequence are $x-5$, $x-1$ and $2x+4$.

(a) Explain why $(x-1)(x-1)=(x-5)(2x+4)$. (2 marks)

common ratio so we can say $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓

$\therefore \frac{x-1}{x-5}=\frac{2x+4}{x-1}$

Cross multiply

$\therefore (x-1)(x-1)=(x-5)(2x+4)$ ✓

(b) Determine the value(s) of $x$. (3 marks)

$x^2-2x+1=2x^2+4x-10x-20$

$0=x^2-4x-21$

$0=(x-7)(x+3)$

$\therefore x=7$ or $x=-3$

(c) Determine all possible values for the fourth term of the sequence. (2 marks)

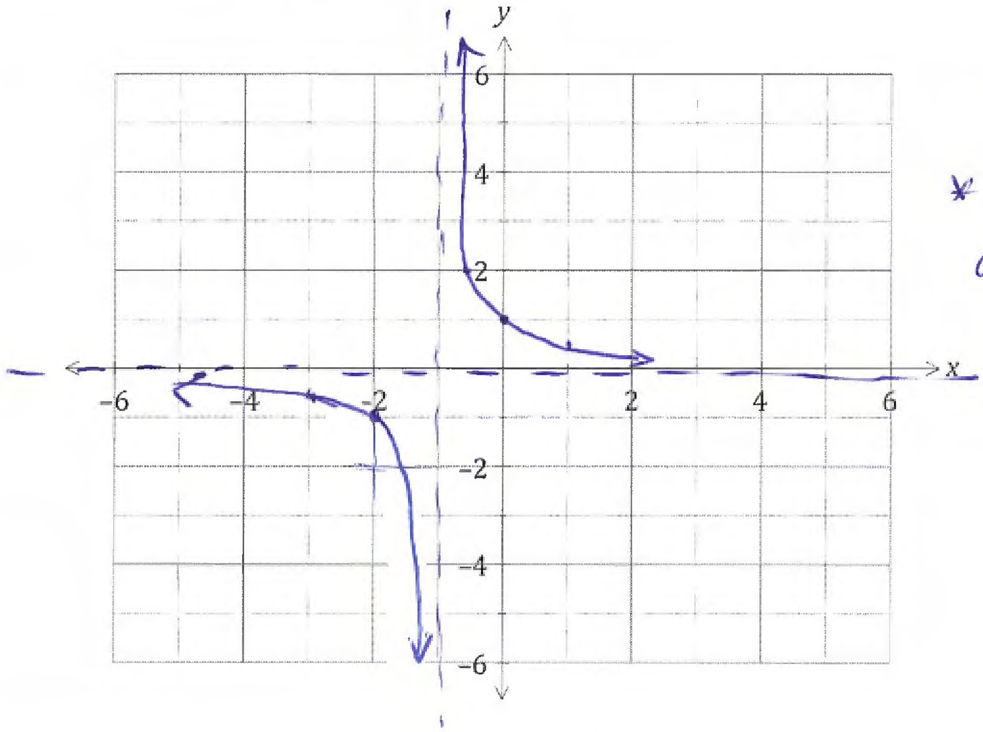
S1 $2, 6, 18, 54$ $\therefore T_4=54$

S2 $-8, -4, -2, -1$ $\therefore T_4=-1$

**Question 8** **(7 marks)**

Let $f(x) = \frac{1}{x+1}$, $x \neq -1$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)




* Must show asymptotes.

(b) Evaluate $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ to determine the slope of $f(x)$ when $x = 2$. (4 marks)

$$\lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \quad @\ x = 2$$

$$= \lim_{h \to 0} \frac{\frac{1}{h+3} - \frac{1}{3}}{h} \quad \checkmark$$

$$= \lim_{h \to 0} \frac{3-(h+3)}{3(h+3)} \div h \quad \checkmark$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{3\cancel{h}(h+3)} \quad \checkmark$$

$$= \frac{-1}{3(3)}$$

$$= -\frac{1}{9} \quad \checkmark$$

**End of questions**

**Additional working space**

Question number: ____________

| Question | Marks Available | Marks Obtained |
|---|---|---|
| 1 | 4 | |
| 2 | 8 | |
| 3 | 4 | |
| 4 | 4 | |
| 5 | 13 | |
| 6 | 5 | |
| 7 | 7 | |
| 8 | 7 | |
| TOTAL | 52 | |
| Weighted Score | 35 | |

SOLUTIONS

# Perth College

**Semester Two Examination, 2016**

**Question/Answer Booklet**

# MATHEMATICS METHODS UNITS 1 AND 2

## Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

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**Section Two: Calculator-assumed** **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

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**Question 9** **(6 marks)**

(a) Determine the equation of the straight line that passes through the point $(8, 11)$ and is perpendicular to the line with equation $2x + 5y = 1$. (3 marks)

$5y = 1 - 2x$

$y = \frac{1-2x}{5}$ ✓ $\therefore m = -\frac{2}{5}$ ✓

$\perp$ $\therefore y = \frac{5}{2}x + c$

$11 = \frac{5}{2}(8) + c$ OR by Calc.

$c = -9$

$\therefore y = \frac{5}{2}x - 9$ ✓

(b) Calculate and use the discriminant to determine the number of solutions to the equation $9x^2 - 24x + 16 = 0$. (3 marks)

$\Delta = b^2 - 4ac$

$= (-24)^2 - 4(9)(16)$ ✓

$= 0$ ✓

$\therefore$ One soln. ✓

**Question 10** **(8 marks)**

A walking club is planning a charity walk from Perth to Esperance. Food and camping supplies are to be set up at each overnight campsite in advance, using a vehicle based in Perth that is just large enough to carry enough for one campsite.

To leave the supplies at the first campsite, the vehicle must travel 40 km. For the second and third campsites, the vehicle must travel 100 km and 160 km respectively, and this pattern continues.

(a) Determine the distances the vehicle will travel to set up campsites four and five. (1 mark)

220, 280 km ✓ R/W

(b) Determine, in simplified form, a rule for the distance, $d$ km, that the vehicle will have to travel to set up campsite $n$. (2 marks)

AP with $a = 40$ & $d = 60$

$\therefore T_n = a + (n-1)d$

$= 40 + (n-1)60$

$= 60n - 20$ ✓ ✓

(c) The vehicle can travel a maximum of 700 km on one tank of fuel. Determine the number of the furthest campsite the vehicle can leave supplies at, using no more than one tank of fuel. (2 marks)

$60n - 20 = 700$ ✓

$n = 12$

$\therefore$ 12th Campsite ✓

(d) If fuel costs 128 cents per litre and the fuel consumption of the vehicle is 9.5 litres per 100 km, determine the total fuel cost to set up the first 20 campsites. (3 marks)

$S_{20} = 12200$ km ✓

$122 \times 9.5 \times 1.28$ ✓

$= \$1483.52$ ✓

**Question 11** **(8 marks)**

Records show that of the 1756 washing machines sold by a retailer during 2015, 464 were deluxe models and the rest were standard. Of all the machines sold, 42 were returned and 31 of these returned machines were standard models.

(a) Determine how many of the standard models were not returned. (2 marks)

| | Deluxe | Standard | TOTAL |
|---|---|---|---|
| Returned | 11 | 31 | 42 |
| Not | 453 | 1261 | 1714 |
| TOTAL | 464 | 1292 | 1756 |

$\therefore$ 1261 ✓

(b) Calculate, to three decimal places, the probability that a randomly chosen machine from those sold

(i) was a standard model. (1 mark)

$\frac{1292}{1756} = 0.736$ ✓

(ii) was returned. (1 mark)

$\frac{42}{1756} = 0.024$ ✓

(iii) was returned given that it was a deluxe model. (2 marks)

$11 ✓ / 464 = 0.024$

✓

(c) Is there any indication that the likelihood of a machine being returned is independent of the model type? Explain your answer. (2 marks)

$P(Del \cap Ret) = 11/1756$
$= 0.006$ ½

$P(Del) \times P(Ret) = \frac{464}{1756} \times \frac{42}{1756}$
$= 0.006$ ½
$(0.00632)$.

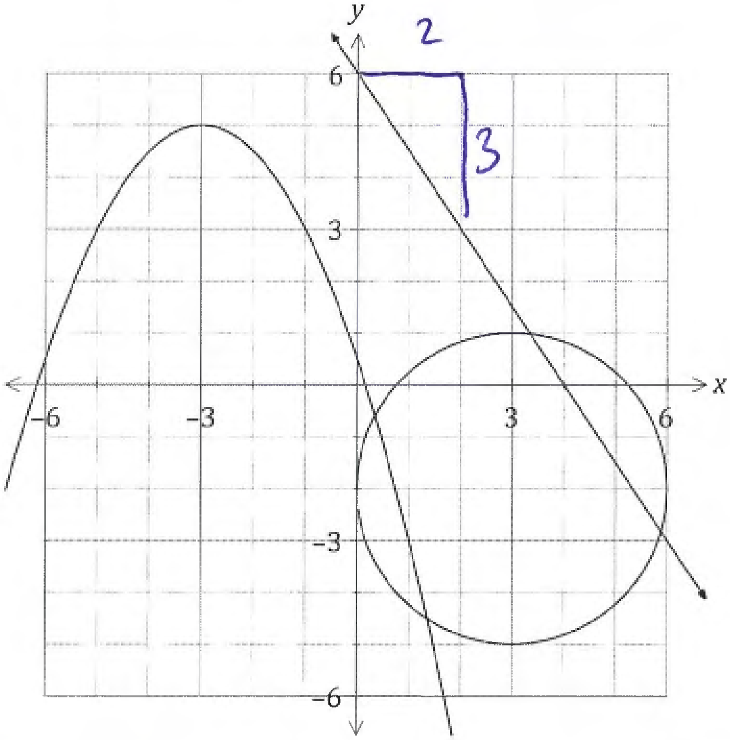
To 3dp these are the same $\therefore$ likely to be independent. ✓

OR $11/1756 \neq 0.00632$
$\therefore$ Not independent.

See next page

**Question 12** **(10 marks)**

The graph of two functions and a circle of radius 3 units are shown.




(a) One function is $f(x) = ax + b$. Determine the values of the constants $a$ and $b$. (2 marks)

$a = -\frac{3}{2}$ ✓

$b = 6$ ✓

(b) The relation can be written in the form $x^2 + px + y^2 + qy + r = 0$.

Determine the values of the constants $p, q$ and $r$. (3 marks)

$(x-3)^2 + (y+2)^2 = 3^2$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 9$

$x^2 - 6x + y^2 + 4y + 13 - 9 = 0$

$\therefore p = -6, \ q = 4, \ r = 4$

✓ ✓ ✓

(c) The other function is $g(x) = cx^2 + dx + e$.

(i) Determine the values of the constants $c, d$ and $e$, given that $g(x)$ has a maximum at $(-3, 5)$. (3 marks)

$g(x) = a(x+3)^2 + 5$

Subst $(-5,3)$ $3 = a(-2)^2 + 5$

$3 = 4a + 5$

$\therefore a = -\frac{1}{2}$

$g(x) = -\frac{1}{2}(x^2 + 6x + 9) + 5$

$= -\frac{1}{2}x^2 - 3x - \frac{9}{2} + 5$ ✓ ✓ ✓

$= -\frac{1}{2}x^2 - 3x + \frac{1}{2}$ $\therefore c = -\frac{1}{2}, d = -3, e = \frac{1}{2}$

(ii) State coordinates of the turning point of the graph of $y = g(x - 7)$. (1 mark)

7 units right

$\therefore (4, 5)$

✓ R/W

(iii) State the range of the function $y = -g(x)$. (1 mark)

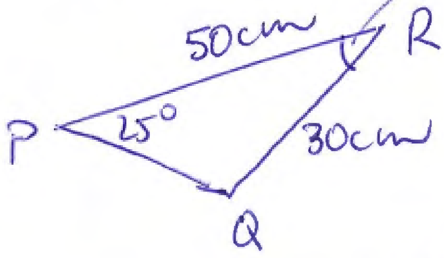
$\{y : y \geq -5, y \in \mathbb{R}\}$

✓

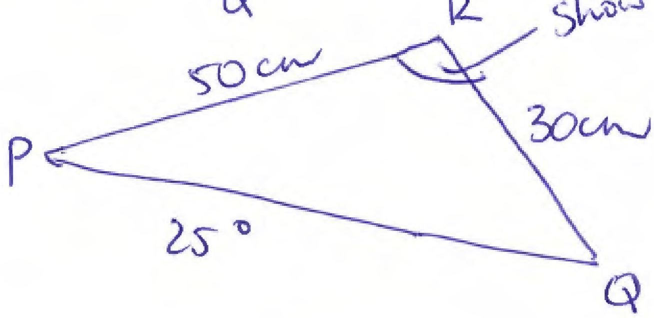
**Question 13** **(6 marks)**

In triangle $PQR$, $PR = 50$ cm, $QR = 30$ cm and $\angle QPR = 25°$.

(a) Sketch two possible triangles that $PQR$ could represent. (Your diagrams do not need to be to scale). (2 marks)




Show acute




Show obtuse

(b) Given that $\angle PQR$ is greater than $75°$ determine

(i) the size of $\angle PQR$. (2 marks)

$$\frac{\sin Q}{50} = \frac{\sin 25}{30}$$

$$Q = 135.2°$$

$$(44.8° < 75°)$$

(ii) the area of triangle $PQR$. (2 marks)

$$\angle R = 19.78°$$

$$\text{Area} = \frac{1}{2}(50)(30)\sin(19.78)$$

$$= 253.81 \text{ cm}^2$$

**Question 14** **(10 marks)**

The function $f$ is given by $f(x) = x^3 - 3x + 2$.

(a) Show that the graph of $y = f(x)$ has two roots and state their coordinates. (2 marks)

$x^3 - 3x + 2 = 0$

$x = 1, \ -2$

i.e. $(1,0)$ ✓ $(-2,0)$ ✓

(b) Use calculus techniques to determine the coordinates of all stationary points of the graph of $y = f(x)$ and use the sign test to determine the nature of these points. (5 marks)

$f'(x) = 3x^2 - 3$ ✓

$3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$ ✓

| $x$ | 0.9 | 1 | 1.5 |
|---|---|---|---|
| $\frac{dy}{dx}$ | − | 0 | + |
| | \ | — | / |

✓

| $x$ | −1.1 | −1 | −0.9 |
|---|---|---|---|
| $\frac{dy}{dx}$ | + | 0 | − |
| | / | — | \ |

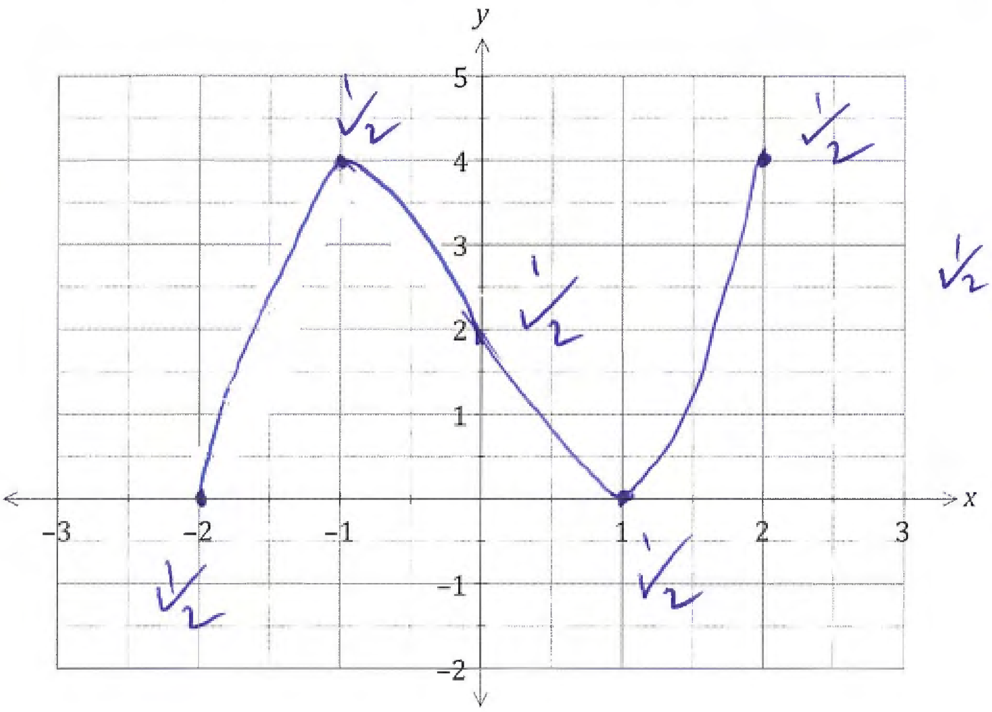
✓

$\therefore (1,0)$ is a min $\frac{1}{2}$ ✓ $\therefore (-1,4)$ is a max $\frac{1}{2}$ ✓

(c) Sketch the graph of $y = f(x)$ on the axes below for $-2 \le x \le 2$. (3 marks)

$f(-2) = 0$

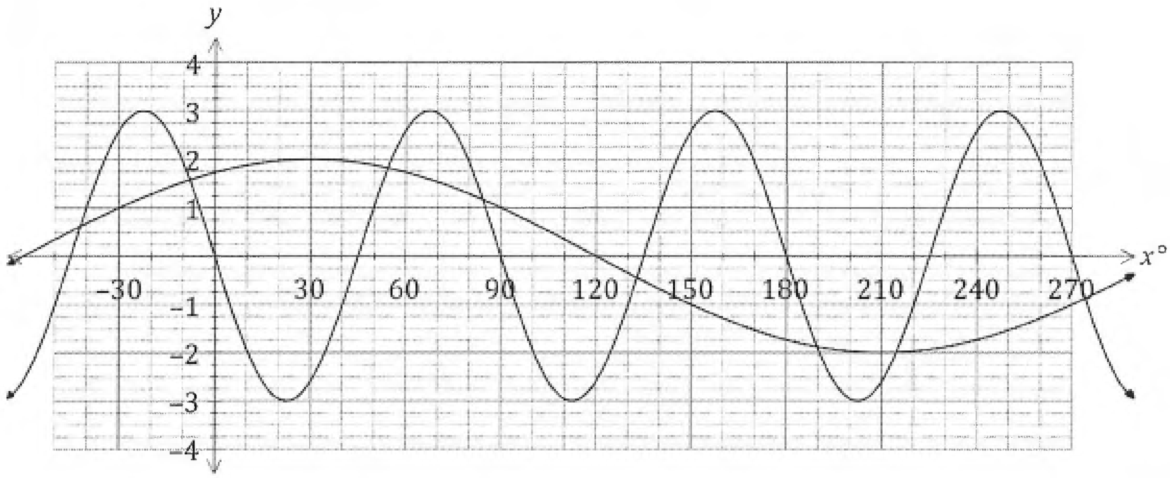
$f(2) = 4$




½ general shape (curved)

**Question 15** **(10 marks)**

(a) The graphs of $f(x) = a\sin(bx)$ and $g(x) = c\cos(x + d)$, where $x$ is in degrees, are shown below.




(i) Determine the values of the constants $a, b, c$ and $d$. (4 marks)

$f(x) = -3\sin 4x$

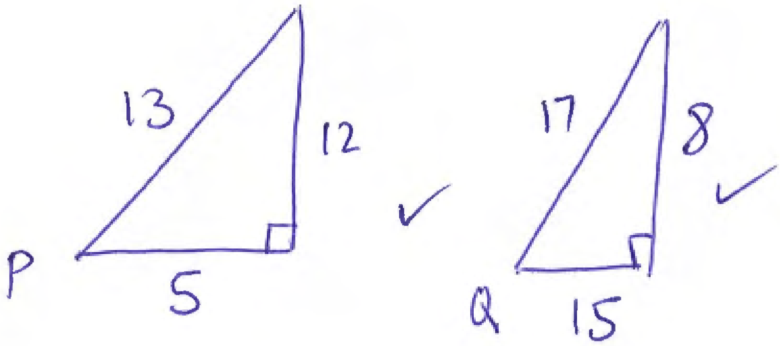
$g(x) = 2\cos(x - 30)$

i.e. $a = -3$ ✓
$b = 4$ ✓
$c = 2$ ✓
$d = -30$ ✓

(ii) Use the graph to solve, to the nearest degree, $f(x) = g(x)$, $0° \leq x \leq 180°$. (2 marks)

$x \simeq 54°$.
$x \simeq 84°$ $(\pm 1°)$.
$x \simeq 133°$
✓✓

(b) $P$ and $Q$ are acute angles with $\sin P = \frac{12}{13}$ and $\cos Q = \frac{15}{17}$. Determine the **exact** value of $\cos(P - Q)$. (4 marks)




$$\cos(P-Q) = \cos P \cos Q + \sin P \sin Q$$

$$= \frac{5}{13} \times \frac{15}{17} + \frac{12}{13} \times \frac{8}{17}$$

$$= \frac{171}{221}$$

**Question 16** **(8 marks)**

The imprisonment rate $R$, in number of prisoners per 100 000 people, in the US between the years 1960 and 2000, can be modelled by the following equation, where $n$ is the year.

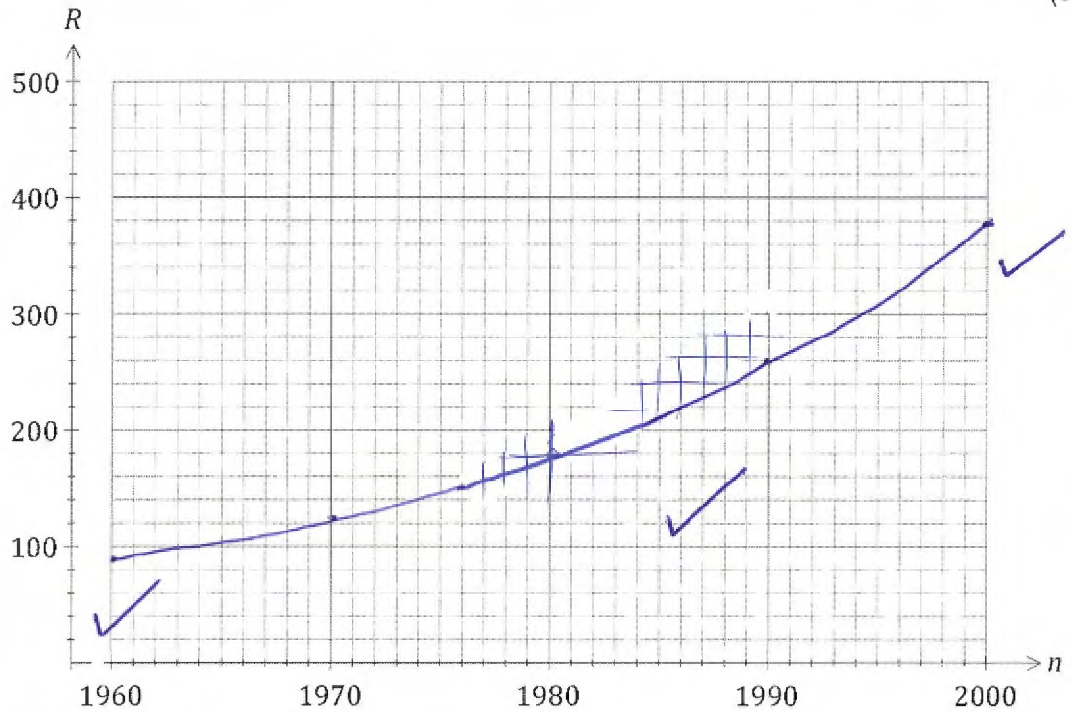
$$R = 85(1.038)^{n-1960}$$

(a) Calculate the imprisonment rate in the year 2000. (1 mark)

$85(1.038)^{40}$ ✓

$= 377.84 \quad \therefore \approx 378$ prisoners/100000 people.

(b) Draw the graph of the imprisonment rate for $1960 \leq n \leq 2000$ on the axes below. (3 marks)




(c) The population of the US was 266 million in 1995. Determine the number of prisoners in the US at this time, to the nearest 1 000. (3 marks)

$R = 85(1.038)^{35}$

$= 313.56407$ ✓

$\frac{266000000}{100000} = 2660$ ✓

$\therefore 2660 \times 313.564$

$= 834\,000$ prisoners (to nearest '000) ✓

(d) When $R$ first exceeded 500, steps were taken to address the exponential growth in the prison population and the model no longer applied. In what year did this occur? (1 mark)

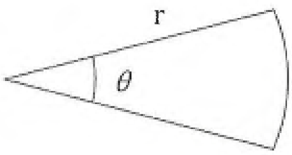
$500 = 85(1.038)^{n-1960}$

$n = 2007.5$

$\therefore$ During 2007 ✓

**Question 17** **(8 marks)**

The perimeter of a sector of a circle, of radius $r$ cm and central angle $\theta$ radians, is 60 cm.




(a) Show that $\theta = \frac{60}{r} - 2$. (2 marks)

$l + 2r = 60 \quad \& \quad l = r\theta$

$\therefore \; r\theta + 2r = 60$ ✓

$r\theta = 60 - 2r$

$\theta = \frac{60 - 2r}{r}$

$\theta = \frac{60}{r} - 2$ ✓

(b) Show that the area of the sector is given by $30r - r^2$. (2 marks)

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} r^2 \left( \frac{60}{r} - 2 \right)$ ✓

$= \frac{1}{2} r \times 60 - r^2$

$= 30r - r^2$ ✓

(c) Use calculus to determine the maximum area of the sector and state the values of $r$ and $\theta$ that achieve this maximum. (4 marks)

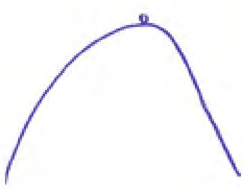
$\frac{dA}{dr} = 30 - 2r$ ✓½

$30 - 2r = 0$

$r = 15$ ✓ $+ \quad \theta = \frac{60}{15} - 2$

$= 2$ ✓

Shape



$\therefore$ Max TP. ✓½

$\therefore$ Max Area $= 30(15) - 15^2$

$= \underline{\underline{225 \text{ cm}^2}}$ ✓

**Question 18** **(8 marks)**

(a) Two students are to be chosen from a class of 18.

(i) Determine how many different pairs of students may be chosen. (1 mark)

$^{18}C_2 = 153$ ✓

(ii) One of the students in the class is the oldest in the school. What is the probability that this student is included in the pair chosen? (2 marks)

$$\frac{^1C_1 \, ^{17}C_1}{^{18}C_2} = \frac{17}{153} = \frac{1}{9}$$ ✓ ✓

(b) A box contains 13 cans of soup, four of which have tomato as an ingredient and the remainder that do not. Four cans are to be selected at random from the box.

(i) Calculate how many different selections of four cans can be made from the box. (1 mark)

$^{13}C_4 = 715$ ✓

(ii) Determine the probability that none of the four cans will have tomato as an ingredient. (2 marks)

$$\frac{^4C_0 \, ^9C_4}{^{13}C_4} = \frac{126}{715} = 0.176$$ ✓ ✓

(iii) Determine the probability that in the selection of four cans, there will be an equal number of cans with and without tomato as an ingredient. (2 marks)

$$\frac{^4C_2 \, ^9C_2}{^{13}C_4} = \frac{216}{715} = 0.302$$ ✓ ✓

**Question 19** **(7 marks)**

(a) A sequence is defined by $T_{n+1} = T_n - 7,\ T_1 = 111$.

(i) Determine $T_{20}$. (1 mark)

$T_{20} = -22$ ✓

(ii) The sum of the first 40 terms, $S_{40}$. (1 mark)

$S_{40} = -1020$ ✓

(iii) The value of $n$ that maximises $S_n$. (2 marks)

$n = 16$ ✓✓

(b) A geometric sequence with $T_2 = 87.5$ has a sum to infinity of 800. Determine all possible values of $T_1$ for this sequence. (3 marks)

$800 = \frac{a}{1-r}$ ✓½ $\qquad ar = 87.5$ ✓½

solve

$\therefore a = 100$, $r = 0.875$ ✓ or $a = 700$, $r = 0.125$ ✓

**Question 20** **(9 marks)**

(a) Show that the equation of the tangent to the curve $y = \dfrac{x + x^3}{2}$ at the point where $x = 2$ is $13x - 2y = 16$. (4 marks)

$\dfrac{dy}{dx} = \dfrac{1}{2} + \dfrac{3x^2}{2}$ ✓ $(2, 5)$

$@ x = 2 \quad \dfrac{dy}{dx} = \dfrac{1}{2} + 6$

$= \dfrac{13}{2}$ ✓

$\therefore$ Eqn is $y = \dfrac{13}{2}x + c$

$5 = 13 + c$ ✓

$c = -8$

$\therefore y = \dfrac{13}{2}x - 8$

$2y = 13x - 16$

$13x - 2y = 16$ $\square$.

(b) The line with equation $y = 5x + c$ is a tangent to the curve $y = x^3 + 3x^2 - 4x - 12$. ✓

Determine the value(s) of $c$. (5 marks)

$\dfrac{dy}{dx} = 3x^2 + 6x - 4$ ✓

$3x^2 + 6x - 4 = 5$

$x = -3 \quad \therefore y = 0$ ✓

$x = 1 \quad \therefore y = -12$ ✓

$\therefore 0 = -15 + c \qquad -12 = 5 + c$

$c = 15 \qquad c = -17$

✓ ✓

**End of questions**